Read the questions carefully and mind the units.

Write your name and student number. This exam comprises 5 problems. The total number of points is 46. The final grade is obtained by dividing the number of points by 4,6.

Problem 1 (10 pts)

a. Consider the nuclear reaction

$$n + {}^{235}_{92}U \longrightarrow {}^{236}_{92}U^{*} \longrightarrow {}^{118}_{46}Pd + {}^{118}_{46}Pd$$

Explain why this is not a realistic fission reaction. (3 pts)

b. Each fission of U-235 is accompanied by the emission of 2 to 3 neutrons. Yet, once established, the chain reaction doesn't runaway and make the reactor supercritical. Describe what happens to the neutrons that do not cause a fission. (4 pts)
c. Delayed neutrons make up less than 1% of all neutrons in a reactor. Explain why they are essential in a reactor. (3 pts)

Solution

a. The distribution of fission fragments is asymmetric, with one lighter fragment and a heavier one.
b. Neutrons that don't cause fissions can be absorbed in the fuel in a (n, γ) capture reaction, can be absorbed in the material of the reactor, can be absorbed in water, creating deuterium $\left({}^{2}H\right)$ or can leak out from the reactor.
c. Delayed neutrons have the effect of raising the average neutron lifetime.

Problem 2 (10 pts)

Consider a critical cubic reactor, with sides equal to 5.4 m. The macroscopic fission cross section is $\Sigma_f = 0.0041\ cm^{-1}$, and the fission reaction rate is $9.91 \times 10^{10} cm^{-3} s^{-1}$.

a. Calculate the power of the reactor in MW. (3 pts)
b. Calculate the maximum neutron flux in the reactor. (4 pts)
c. You have at your disposal fuel rods with two different enrichments (high and low). How would you arrange the fuel so that the flux is more or less the same over the whole reactor? (3 pts)

Solution

a. The power of the reactor is

$$P = E_f R_f V = 200 \times 1{,}602\ 10^{-13} \times 9.91 \times 10^{10} \times 540^3 = 500\ Js^{-1} = 500\ MW$$

b. The maximum neutron flux is found at the centre of the reactor $(0,0,0)$. It is given by the expression:

$$\phi_{max} = A\ \cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{b}y\right)\cos\left(\frac{\pi}{c}z\right)$$

Since $(x, y, z) = (0,0,0)$, the maximum flux is $\phi_{max} = A$.

$$\phi_{max} = A = 3.87 \frac{P}{V E_f \Sigma_f}$$

$$\phi_{max} = 3.87 \frac{500\ 10^6}{540^3 \times 200 \times 1.602\ 10^{-13} \times 0.0041} = 9.35\ 10^{13} cm^{-2} s^{-1}$$

c. Since the neutron flux is maximum at the centre of the reactor, we need to put low enrichment fuel in the centre and higher enrichment fuel towards the sides of the reactor. This would flatten the flux.

Problem 3 (10 pts)

A thermal reactor contains $m = 150$ tonnes of natural uranium (containing 0.72% of U-235) and runs with a neutron flux $\phi = 10^{13}\ cm^{-2}s^{-1}$. The microscopic fission and neutron capture cross sections for U-235 are $\sigma_f = 579\ b$ and $\sigma_{n,\gamma} = 101\ b$, respectively.

a. Determine the power P. (3 pts)
b. Calculate the absorption rate of U-235 (in s^{-1}). (4 pts)
c. How much of the initial U-235 load is consumed in a year? (3 pts)

Solution

a. The power per cm^3 is given by

$$P = E_f R_f = E_f \Sigma_f \phi = E_f \sigma_f N \phi$$

We first need to calculate the number of U-235 atoms in the fuel.

$$N = m \frac{N_A}{M} = 150\ 10^6 \times 0.0072 \times \frac{6.022 \times 10^{23}}{238} = 2.73\ 10^{27}\ atoms$$

The power is then

$$P = E_f \sigma_f N \phi = 200 \times 1.602\ 10^{-13} \times 579\ 10^{-24} \times 2.73\ 10^{27} \times 10^{13} = 506\ MW$$

b. The absorption rate of U-235 is due to both fission and neutron capture. The absorption cross section is $\sigma_a = \sigma_f + \sigma_{n,\gamma} = 579 + 101 = 680\ b = 680\ 10^{-24}\ cm^2$.

The absorption rate of U-235 is given by

$$N \sigma_a \phi = 2.73\ 10^{27} \times 680\ 10^{-24} \times 10^{13} = 1.86\ 10^{19}\ s^{-1}$$

c. The number of U-235 atoms consumed in a year is

$$N \sigma_a \phi\ (365 \times 24 \times 3600) = 1.86\ 10^{19} (365 \times 24 \times 3600) = 5.85\ 10^{26}\ atoms$$

The initial load of U-235 is $2.73\ 10^{27}\ atoms$, and $5.85\ 10^{26}/2.73\ 10^{27} = 0.21$, that is 21% of the U-235 has been consumed in a year.

Problem 4 (8 pts)

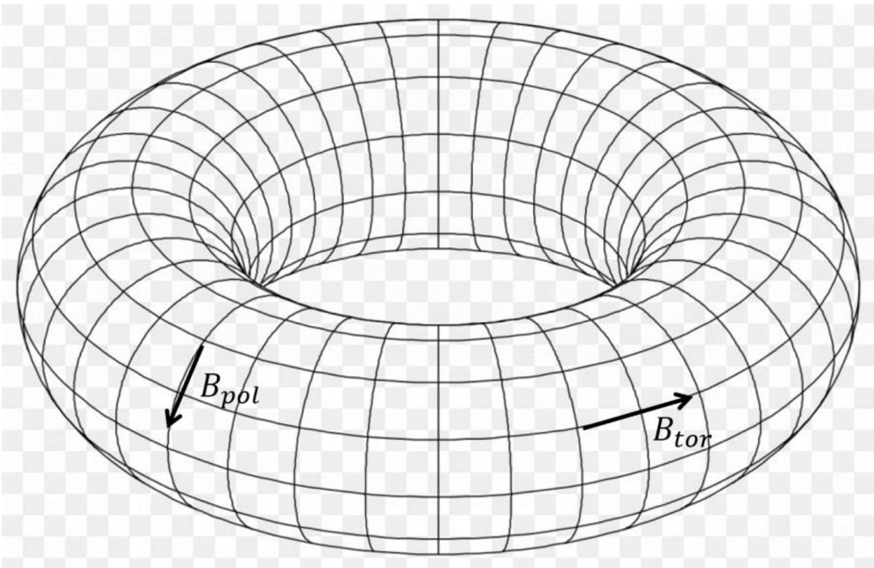
Consider a fusion power plant built around a tokamak, where a D-T plasma is confined in the vessel by magnetic fields.

a. Use the figure of the torus to draw the toroidal and poloidal magnetic fields. (2 pts)
b. How is the poloidal magnetic field generated in the tokamak? (3 pts)
c. The fusion reaction inside the plasma is

$$D + T \rightarrow {}^{4}He + n$$

Explain why it is crucial to stop neutrons in the blanket? (3 pts)

Solution

a. 

b. The poloidal magnetic field is generated by the current induced by the solenoid. Additional poloidal magnets around the tokamak also add to the magnetic field.

c. To increase the efficiency of producing electricity, we want to capture most of the neutron heat. Neutrons transfer their heat colliding with the material, progressively slowing down. To protect the superconductors: if not stopped, neutrons risk to damage or heat the magnets behind the blanket. To be superconductive magnets must stay cool!

Problem 5 (8 pts)

Imagine that in a few years from now, you are put in charge of the nuclear energy R&D (research and development) programme of your country. Recalling (fondly) your course on nuclear energy at the University of Groningen, you realise that several choices are available, including getting rid of the nuclear option altogether.

In your proposal to the funding agency, you highlight and justify the activities you would like to pursue and/or start in the short-term (5 years). Keep your writing to one page.

Avogadro number $N_A = 6.022 \times 10^{23} mol^{-1}$

Cross section $1\ b = 10^{-24} cm^2$

Number density

$$N\ [cm^{-3}] = \frac{\rho\ [g/cm^3]\ N_A\ [mol^{-1}]}{M\ [u = g/mol]}$$

Reaction rate $R = \Sigma\phi = N\sigma\phi$

Total average power of a reactor

$$P[W = J/s] = E_f[J] \times R_f[cm^{-3}s^{-1}] \times V[cm^3]$$

Or

$$P[W = J/s] = E_f[J] \times R_f[s^{-1}]$$

$1\ MeV = 1.602\ 10^{-13}\ J$ and $1\ W = 1\ J/s$

Buckling and neutron flux for different reactor geometries

Geometry	**Dimensions**	**Buckling B^2**	**Flux**	**A**
Infinite slab	Thickness a	$\left(\frac{\pi}{a}\right)^2$	$A\cos\left(\frac{\pi}{a}x\right)$	$1.57 \times \frac{P}{a\ E_f\ \Sigma_f}$
Rectangular parallepiped	a x b x c	$\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$	$A\cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{b}y\right)\cos\left(\frac{\pi}{c}z\right)$	$3.87 \times \frac{P}{V\ E_f\ \Sigma_f}$
Infinite cylinder	Radius R	$\left(\frac{2.405}{R}\right)^2$	$AJ_0\left(\frac{2.405\ r}{R}\right)$	$0.738 \times \frac{P}{R^2 E_f \Sigma_f}$
Finite cylinder	Radius R, height H	$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$	$AJ_0\left(\frac{2.405\ r}{R}\right)\cos\left(\frac{\pi\ z}{H}\right)$	$3.63 \times \frac{P}{V\ E_f\ \Sigma_f}$
Sphere	Radius R	$\left(\frac{\pi}{R}\right)^2$	$A\ \frac{1}{r}\sin\left(\frac{\pi\ r}{R}\right)$	$\frac{P}{4R^2\ E_f\ \Sigma_f}$